New Method for Online Frequency Response Function Estimation Using Circular Queue

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ABSTRACT

Input-Output relationship of the system is estimated using frequency response function (FRF) model. These estimation algorithms are generally implemented for off-line identification using MATLAB but in this paper, it has been used to perform on-line identification. H₀ estimation algorithm can be used to real time calculation of frequency response function. H₀ estimator identifies the system FRF which is then combined with Fast Fourier Transform (FFT) and inverse Fast Fourier Transform (IFFT) technique to correct the input. The error of the expected output signal and the experiment output signal is calculated and corrected using FFT, IFFT and FRF to compensate the input signal.

Keywords — Fast Fourier transform, Frequency response function, Circular array.

1. INTRODUCTION

At present, as a method of establishing mathematical model of the system, the system identification has been widely applied to the automatic control, aviation, spaceflight, astronomy, medicine, biology, marine ecology and society, economics and many other fields. With the rapid development of science and technology, the status of system identification technique in various disciplines is becoming increasingly important.

System identification is a method of using the input and output data of the system to build the mathematical model of the system, which is being widely applied to various fields of production and life. Since modern systems are becoming more and more complex, and even their internal mechanisms are unclear in whole, therefore, whether or not researchers design a rational system identification method to establish the mathematical model of the system plays an important role in the process of system identification.

Each identification method consists of a series of basic steps. Some of them may be hidden or selected without the user being aware of his choice. Clearly, this can result in poor or sub optimal results.

In each session the following actions should be taken:

- Collect information about the system.
- Select a model structure to represent the system.
- Estimate the model parameters to fit the model as well as possible to the measurements.
- Validate the selected model.

2. FREQUENCY RESPONSE FUNCTION

In model formulation, input data is selected for which theoretical modelling is to be done. It is very important and the important fact is to keep in mind that information not contained in data does not appear in the model. After model formulation next step is parameter estimation. Parameter estimation can be done in both frequency domain and time domain. After estimation the parameter estimation, validation of the model is done. The response of system is compared with expected response and this process is iteratively performed until exact response is found.

The methods can be classified into: (i) parametric and (ii) non-parametric methods. In a parametric model, the system is described using limited number of characteristic quantities.
called the parameters of the model, while in a nonparametric model the system is characterized by measurements of a system function at a large number of points. Examples of parametric models are the transfer function of a filter described by its poles and zeros, the motion equations of a piston, etc. An example of a non-parametric model is the description of a filter by its impulse response at a large number of points.

Non-parametric system identification methods are divided into time domain and frequency domain methods. Frequency domain methods are again classified by the input signal used for identification method as (i) Impulse response function, (ii) Step response function and (iii) Frequency response function. Impulse response uses Impulse signal as input to system to be identified while Step response function uses Step signal as input to the system. Frequency response function is a mathematical representation of the relationship between the input and the output of a system.

Frequency response functions (FRFs) are very important in many engineering applications, and they provide particularly useful information about an unknown linear system. They can be estimated either through the Fourier transforms of the input–output cross-correlation function and the input autocorrelation function or directly from the Fourier transforms of the input–output records. When the input signal is random, the FRF estimates are biased, with a significant variance, even in the absence of any measurement noise.

Frequency response functions are normally used to describe input-output (force-response) relationships of any system. Most often, the system is assumed to be linear and time invariant although this is not necessary. In the cases where assumptions of linearity and time invariance are not valid, the measurement of frequency response functions are also dependent upon the independent variables of time and input.

The estimation of the frequency response function depends upon the transformation of data from the time to frequency domain. The Fourier transform is used for this computation. Unfortunately, though, the integral Fourier transform definition requires time histories from negative to positive infinity. Since this is not possible experimentally, the computation is performed digitally using FFT algorithm which is based upon limited time history.

The most common approach to the estimation of frequency response functions is by use of least square (LS) or total least square (TLS) techniques. This is a standard technique for estimating parameters in the presence of noise. LS methods minimize the square of the magnitude error and thus compute the best estimate of the magnitude of the FRF but have little effect on the phase of the FRF. The primary difference between the algorithms used to estimate frequency response functions is in the assumption of where the noise enters the measurement problem.

Three algorithms, referred to as H₁, H₂ and Hₐ algorithms are commonly available for estimating frequency response functions. Table 1 summarizes the characteristics for the algorithms.

Table 1 Summery of Frequency Response Function Models

<table>
<thead>
<tr>
<th>Technique</th>
<th>Solution</th>
<th>Assumed Location of Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₁</td>
<td>LS</td>
<td>no noise</td>
</tr>
<tr>
<td>H₂</td>
<td>LS</td>
<td>noise</td>
</tr>
<tr>
<td>Hₐ</td>
<td>TLS</td>
<td>noise</td>
</tr>
</tbody>
</table>

For system identification, H₁ algorithm is used. This algorithm assumes noise at the output of the system. The frequency response function is given by

\[ H = \frac{S_{xy}}{S_{xx}} \]  

Where, \( S_{xx} \) = Auto power spectrum of \( x \).
\( S_{xy} \) = Cross power spectrum of \( x \) and \( y \).

\[ S_{xy} = FFT(y) \ast FFT^*(x) \]  

\[ S_{xx} = FFT(x) \ast FFT^*(x) \]  

3. CIRCULAR QUEUE

A circular buffer is a reserved part of memory which serves as a temporary buffer for data. It is organized in a special manner: the incoming data fills the buffer until it is completely full, and then starts to fill the buffer again from the beginning overwriting the previous content. It is the task of the software to make use of the stored data before it gets overwritten by new data. The use of Circular queue reduces the memory required for implementation of the proposed algorithm.
4. PROPOSED ALGORITHM

Generally, these algorithms are computed in off-line fashion but for real time system, implementation is required to be done online. The algorithm implementation is done such that FRF is updated after each 10 milliseconds.

Consider X(n) be an input to servo hydraulic actuator, Y(n) be an output from the position sensor and R(n) be an expected output from actuator. All 3 parameters are stored in separate 512 float array. Input X(n) is applied at each millisecond and Output Y(n) is also acquired at each millisecond. R(n) is a set-point file stored in the memory card. R(n) consists of a sine-sweep set-point data table generated using a computer program. The amplitude of sine-sweep signal is 1 mm. The frequency of sine-sweep signal is chosen from 0 Hz to 50 Hz.

Consider R(n) has 10000 samples of set-point data as shown in figure 1

![Figure 1 R(n) before 1st iteration](image)

The shaded portion i.e. for the first 522 milliseconds, R(n) is applied as it is to the actuator. This is required to be done as initially Y(n) array does not contain any data. For this amount of time, conventional PID controller can be used for the controlling the actuator and float array X(n) and Y(n) gets accumulated till 512 points.

![Figure 2 X(n) before 1st iteration](image)

![Figure 3 Y(n) before 1st iteration](image)

512 point FFT algorithm is applied to X(n) and Y(n)

FFT of X

\[ X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi kn}{N}} \] ..................(4)

FFT of Y

\[ Y(k) = \sum_{n=0}^{N-1} y(n) \cdot e^{-j\frac{2\pi kn}{N}} \] ..................(5)

The output of FFT is 512 float data. Middle 10 data point is applied as next input to the servo actuator. Figure 4 shows the input data array after first iteration of algorithm. The new 10 points of X(n) array is shown in blue colour. This algorithm is recursively run until all set-points are completed.

![Figure 4 X(n) after 1st iteration](image)

5. CONCLUSIONS

System identification algorithm is generally implemented for off-line identification using MATLAB. A new method has been developed which is used to perform on-line identification. Non-parametric system identification method is chosen because of its inherent advantage of being faster than parametric system identification method. Parametric identification also requires prior knowledge of system to be identified.

Identification in the frequency domain makes use of the Fast Fourier Transform algorithm. The Fast Fourier Transform (FFT) algorithm reduces the number of complex
multiplications required in the evaluation of the Fourier Transform and provides a means of performing system identification on-line or approximately in real time.

Frequency response function gives input-output relationship of the system. It is implemented using H₁ algorithm which assumes location of noise at the output of system and reduces noise using least square estimation.

REFERENCES


